As in the case of the ordinary Markov process, a question of great significance is that of determining the asymptotic behavior as $n \to \infty$. It is reasonable to suspect, from the nature of the underlying decision process, that a certain steady-state behavior exists as $n \to \infty$. This can be established in a number of cases.

The author does not discuss these matters at all. This is unfortunate, since there is little value to steady-state analysis unless one shows that the dynamic process asymptotically approaches the steady-state process as the length of the processes increases. Furthermore, it is essential to indicate the rate of approach.

The author sets himself the task of determining steady-state policies under the assumption of their existence. Granted the existence of a "steady state," the functions $f_i(n)$ have the asymptotic form $nc + b_i + o(1)$ as $n \to \infty$, where c is independent of i. The recurrence relations then yield a system of equations for c and the b_i.

This system can be studied by means of linear programming as a number of authors have realized; see, for example, A. S. Manne, "Linear programming and sequential decisions," *Management Science*, vol. 6, 1960, p. 259–268.

Howard uses a different technique based upon the method of successive approximations, in this case an approximation in policy space. It is a very effective technique, as the author shows, by means of a number of interesting examples drawn from questions of the routing of taxicabs, the auto replacement problem, and the managing of a baseball team.

The book is well-written and attractively printed. It is heartily recommended for anyone interested in the fields of operations research, mathematical economics, or in the mathematical theory of Markov processes.

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55[V, X].—GERTRUDE BLANCH, KARL GOTTFRIED GUDERLEY & EMMA MARIAN VALENTINE, *Tables Related to Axial Symmetric Transonic Flow Patterns*, WADC Technical Report 59–710, 1960, Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C., xlviii + 108 p., 27 cm.

The equations of motion of a compressible fluid are non-linear and are generally difficult to handle. In certain cases, such as in the flow past slender bodies of revolution, the equations can be approximated by much simpler ones. For subsonic and supersonic flow these approximating equations are linear. When the flow velocity is nearly equal to one, the approximate equation for the disturbance potential takes the non-linear form

$$-\Phi_x \Phi_{xx} + \Phi_{yy} + \frac{\Phi_y}{y} + \frac{1}{y^2} \Phi_{\omega\omega} = 0$$

when x, y, ω are cylindrical coordinates. K. G. Guderley and his colleague H. Yoshihara have studied the flow past slender bodies at Mach numbers close to one in a series of papers and in a book by Guderley, *Theorie schallnaher Strömungen*, Springer-Verlag, 1957.

The basic technique applied to axially symmetric flows is to find a basic solution Φ^B of the form

$$\Phi^B = y^{3n-2}f(\zeta)$$

where $\zeta = x/y^n$ and n is a constant. The variable f then satisfies the ordinary differential equation

$$(f' - n^2\zeta^2)f'' + (5n^2 - 4n)\zeta f' - (3n - 2)^2 f = 0.$$

Further solutions necessary to satisfy particular boundary conditions are then found by perturbing the basic solution by the function $\bar{\Phi}$, i.e.,

$$\Phi = \Phi^B + \bar{\Phi}(x, y, \omega).$$

Then $\bar{\Phi}$ is assumed to satisfy the linear equation

$$-\Phi_x{}^B\bar{\Phi}_{xx} - \Phi_{xx}^B\,\bar{\Phi}_x + \bar{\Phi}_{yy} + \frac{\bar{\Phi}_y}{y} + \frac{\bar{\Phi}_{\omega\omega}}{y^2} = 0.$$

Particular solutions of the equation are then found in the form

$$\bar{\Phi} = y^\nu g(\zeta)\cos m\omega; \qquad m = 0, 1, 2, \cdots$$

Then $g(\zeta)$ satisfies the ordinary differential equation

$$(f' - n^2\zeta^2)g'' + [f'' + (2\nu n - n^2)\zeta]g' + (m^2 - \nu^2)g = 0.$$

The solution of this equation leads to an eigenvalue problem with the eigenvalue ν.

The present report tabulates the functions f and g together with their derivatives and some other related functions. In Table 1 appear 6D values of $f(\zeta)$ and $dg/d\zeta$ for $\zeta = -7.5(.1) - 3(.02)1$; in Table 2 similar information appears for $g(\zeta)$ and $dg/d\zeta$. These tabular data are given for several values of the eigenvalue ν.

A rather complete discussion of the mathematical problems involved is given in the introduction. The eigenvalues are found using a contour integration technique. It is stated that the numerical calculations are performed on an ERA 1103, with considerable pains taken to insure accuracy. The entries are stated to be correct to within one unit in the last place.

The tables should be quite useful to anyone interested in the study of special cases of transonic flow.

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56[W].—H. L. Toothman, *A Table of Probability Distributions useful in War Games and other Competitive Situations*, NRL Report 5480, U. S. Naval Research Laboratory, Washington, D. C., May 16, 1960, i + 91 p., 27 cm.

A player makes a maximum of $(2r - 1)$ plays. On odd-numbered plays he scores 1 with probability p_1 and 0 with probability $1 - p_1$; on even-numbered plays he is eliminated from subsequent play with probability p_2. The probability that he will score exactly n is

$$S_n = \binom{r}{n} p_1^n(1 - p_1)^{r-n}(1 - p_2)^{r-1} + \sum_{k=n}^{r-1} \binom{k}{n} p_1^k(1 - p_1)^{k-n}p_2(1 - p_2)^{k-1}.$$